MATH 118: Practice Midterm 1 Key

Name: ______________________________

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		**90**

1. Short answer questions:

   (a) True or False: We are allowed to use exponent laws in the following way:

$$\left(\frac{x^3y^2}{z^3}\right)^2 = \frac{x^5y^4}{z^5}$$

   $\boxed{\textit{False}.}$

   LoE #3 was violated two times.

   (b) True or false: We can simplify

$$\frac{(x+3)\left[x-2(x+1)\right]}{(x+3)^2}$$

   by crossing out the $x+3$.

   $\boxed{\textit{True}.}$

   $(x+3)$ is a global factor in the numerator and denominator.

   (c) True or false: If a is a real number, it is possible for $|a|$ to be negative.

   $\boxed{\textit{False}.}$

2. Factor and simplify:

(a) $8x^2 + 8x - 6$

Three term. Use GCF first, pull out 2:

$$8x^2 + 8x - 6 = 2(4x^2 + 4x - 3) \qquad \text{GCF}$$
$$= \boxed{2(2x-1)(2x+3)} \qquad \text{new X method}$$

$a = 4,\ b = 4,\ c = -3$

(1) For a

$\begin{array}{c|c|c} 1 & 4 & 2 \\ 4 & 1 & 2 \end{array}$

(2) For b

$\begin{array}{c|c|c|c} -3 & 1 & -1 & 3 \\ 1 & -3 & 3 & -1 \end{array}$

(3) Check against b.

Pick a diagonal product close to b.

$\begin{array}{cc} 4 & -3 \\ 1 & 1 \end{array} \rightarrow 4 - 3 = 1 \neq b$

failed.

product is close to b.

$\begin{array}{cc} 2 & -1 \\ 2 & 3 \end{array} \longrightarrow 2 \cdot 3 + (-1) \cdot 2 = 4 = b$ ✓

close to b.

(b) $(x+3)^2(x-2) + (x+3)(x-2)^2$

Two term. Use GCF first, common factor is $(x+3)(x-2)$.

$$(x+3)^2(x-2) + (x+3)(x-2)^2 = (x+3)(x-2)\left[(x+3) + (x-2)\right] \qquad \text{GCF}$$
$$= (x+3)(x-2)(x+3+x-2) \qquad \text{Drop parentheses}$$
$$= \boxed{(x+3)(x-2)(2x+1)} \qquad \text{Like terms}$$

3. Expand and simplify:

(a) $(3x-1)(4x+5)-2$

Two terms. Focus on the first term as a subproblem.

$$\begin{aligned}
(3x-1)(4x+5)-2 &= (3x-1)4x+(3x-1)5-2 && \text{Dist. Law}\\
&= 12x^2-4x+15x-5-2 && \text{Dist. Law}\\
&= \boxed{12x^2+11x-7} && \text{Like terms}
\end{aligned}$$

(b) $2(2x-1)^2-(2x-1)2x$

Two terms. Focus on each term as a subproblem.

$$\begin{aligned}
2(\overset{A}{2x}-\overset{B}{1})^2-(2x-1)2x &= 2(\overbrace{4x^2-4x+1}^{A^2\ -2AB\ +\ B^2})-4x^2+2x && (A-B)^2,\ \text{Dist. Law}\\
&= 8x^2-8x+2-4x^2+2x && \text{Dist. Law}\\
&= 4x^2-6x+2 && \text{Like terms}\\
&= 2(2x^2-3x+1) && \text{Factor! GCF}\\
&= \boxed{2(2x-1)(x-1)} && \text{new X method}
\end{aligned}$$

4. Simplify:

(a) $\dfrac{1}{x+1} - \dfrac{x}{x-1}$

Subtraction of fractions. Find the LCD.

$$\begin{aligned}
\frac{1}{x+1} - \frac{x}{x-1} &= \frac{x-1}{(x-1)} \cdot \frac{1}{(x+1)} - \frac{x}{(x-1)} \cdot \frac{x+1}{(x+1)} && \text{Introduce what's missing for LCD} \\
&= \frac{x-1}{(x-1)(x+1)} - \frac{x(x+1)}{(x-1)(x+1)} && \text{Frac. Prop. 1} \\
&= \frac{x-1-x(x+1)}{(x-1)(x+1)} && \text{Frac. Prop. 3} \\
&= \frac{x-1-x^2-x}{(x-1)(x+1)} && \text{Dist. Law} \\
&= \frac{-1-x^2}{(x-1)(x+1)} && \text{Like terms} \\
&= \frac{-(1+x^2)}{(x-1)(x+1)} && \text{GCF, factor out negative} \\
&= \boxed{-\frac{x^2+1}{(x-1)(x+1)}} && \text{Put negative in front of fraction}
\end{aligned}$$

(b) $\dfrac{3(x+h)^2-1-(3x^2-1)}{h}$

Think of the numerator and denominator as separate problems.

Denominator is already simplified. Focus on the numerator as an expansion problem, as like terms will be created.

$$\begin{aligned}
\frac{3(\overset{A}{x}+\overset{B}{h})^2-1-(3x^2-1)}{h} &= \frac{3(\overbrace{x^2+2xh+h^2}^{A^2\ +\ 2AB\ +\ B^2})-1-3x^2+1}{h} && (A+B)^2,\ \text{Dist. Law}\\
&= \frac{3x^2+6xh+3h^2-1-3x^2+1}{h} && \text{Dist. Law}\\
&= \frac{6xh+3h^2}{h} && \text{Like terms}\\
&= \frac{h(6x+3h)}{h} && \text{GCF}\\
&= \boxed{6x+3h} && \text{Frac. Prop. 5}
\end{aligned}$$

5. Simplify:

(a) $-8^{\frac{1}{3}}$

Here, the negative is a factor of (-1) attached to the 8.

$$\begin{aligned}
-8^{\frac{1}{3}} &= (-1)\cdot 8^{\frac{1}{3}} && \text{Definition of negative}\\
&= (-1)\cdot \sqrt[3]{8} && \text{Definition of fractional exponent}\\
&= (-1)\cdot \sqrt[3]{2^3} && \text{Prime factorization of 8}\\
&= (-1)\cdot 2^{3/3} && \text{Definition of fractional exponent}\\
&= \boxed{-2}
\end{aligned}$$

(b) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$

$$\begin{aligned}
\left(\frac{4}{9}\right)^{-\frac{1}{2}} &= \frac{9^{\frac{1}{2}}}{4^{\frac{1}{2}}} && \text{LoE \# 7}\\
&= \frac{\sqrt{9}}{\sqrt{4}} && \text{Definition of fractional exponent}\\
&= \boxed{\frac{3}{2}} && \text{Definition of square root}
\end{aligned}$$

(c) $\dfrac{\sqrt[4]{x^3}\cdot x}{x^2}$

Convert roots to exponents and use Laws of Exponents.

$$\begin{aligned}
\frac{\sqrt[4]{x^3}\cdot \cancel{x}}{\cancel{x^2}x} &= \frac{x^{\frac{3}{4}}}{x} && \text{Definition of fractional exponent, Frac. Prop. 5}\\
&= x^{\frac{3}{4}-1} && \text{LoE \# 2}\\
&= x^{\frac{3}{4}-\frac{4}{4}} && \text{LCD}\\
&= x^{-\frac{1}{4}} && \text{algebra}\\
&= \boxed{\frac{1}{x^{\frac{1}{4}}}} && \text{Remove all negative exponents}
\end{aligned}$$

6. Simplify:

$$\frac{\dfrac{1}{x+h+1}-\dfrac{1}{x+1}}{h}$$

Compound fraction problem. Numerator and denominator are separate problems.

Denominator already simplified. Numerator is fraction subtraction problem. Focus on simplifying numerator.

$$\frac{\dfrac{1}{x+h+1}-\dfrac{1}{x+1}}{h} = \frac{\dfrac{x+1}{(x+1)}\cdot\dfrac{1}{(x+h+1)}-\dfrac{1}{(x+1)}\cdot\dfrac{x+h+1}{(x+h+1)}}{h} \quad \text{LCD in numerator}$$

$$= \frac{\dfrac{x+1}{(x+1)(x+h+1)}-\dfrac{x+h+1}{(x+1)(x+h+1)}}{h} \quad \text{Frac. Prop. 1}$$

$$= \frac{\dfrac{x+1-(x+h+1)}{(x+1)(x+h+1)}}{h} \quad \text{Frac. Prop. 3}$$

$$= \frac{\dfrac{x+1-x-h-1}{(x+1)(x+h+1)}}{h} \quad \text{Dist. Law}$$

$$= \frac{\dfrac{-h}{(x+1)(x+h+1)}}{h} \quad \text{Like terms}$$

$$= \frac{-h}{(x+1)(x+h+1)}\cdot\frac{1}{h} \quad \text{Frac. Prop. 2}$$

$$= \frac{-\cancel{h}}{(x+1)(x+h+1)\cancel{h}} \quad \text{Frac. Prop. 1}$$

$$= \frac{-1}{(x+1)(x+h+1)} \quad \text{Frac. Prop. 5}$$

$$= \boxed{-\frac{1}{(x+1)(x+h+1)}} \quad \text{Negative in front}$$

7. If

$$a = x - 1 \qquad b = x + 1 \qquad c = x^2 - 1$$

Calculate and simplify $a^2 - b \cdot c$.

Do not forget parentheses when substituting and subtracting ≥ 2 terms.

This problem is testing you on remembering them due to the above criteria.

$$\begin{aligned}
a^2 - b \cdot c &= (\overset{A}{x} - \overset{B}{1})^2 - (x+1)(x^2-1) && \text{Substitution} \\
&= \overbrace{x^2 - 2x + 1}^{A^2 \;-\; 2AB \;+\; B^2} - \overbrace{\left[(x+1)x^2 - (x+1)\cdot 1\right]}^{\text{subtracting 2 local terms}} && \text{Expand: } (A-B)^2, \text{ Dist. Law} \\
&= x^2 - 2x + 1 - (x^3 + x^2 - x - 1) && \text{Dist. Law} \\
&= x^2 - 2x + 1 - x^3 - x^2 + x + 1 && \text{Dist. Law} \\
&= \boxed{-x^3 - x + 2} && \text{Like terms}
\end{aligned}$$

8. Perform the indicated instruction.

   (a) Rationalize the numerator and simplify: $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

   Two term rationalization problem. Multiply by the conjugate radical.

$$\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{\overset{A}{\sqrt{x+h}}-\overset{B}{\sqrt{x}}}{h}\cdot\frac{\overset{A}{\sqrt{x+h}}+\overset{B}{\sqrt{x}}}{(\sqrt{x+h}+\sqrt{x})} \qquad \text{To use } A^2-B^2$$

$$= \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})} \qquad A^2-B^2$$

$$= \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} \qquad \text{Squaring a square root}$$

$$= \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} \qquad \text{Like terms}$$

$$= \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}} \qquad \text{Frac. prop. 5}$$

   (b) List all possible terms/factors and their context levels for

$$3y-3(x^2y+xy)-6xy(3x^2-1)^2$$

   At each context level, zoom into each underlined term/factor. Perform term/factor analysis on that zoomed context.

   T for term, F for factor.

Context level	Terms/Factors
Global (L1)	$\underbrace{3y}_{T}-\underbrace{3(x^2y+xy)}_{T}-\underbrace{6xy(3x^2-1)^2}_{T}$
Local (L2)	$\underbrace{3}_{F}\underbrace{y}_{F}-\underbrace{3}_{F}\underbrace{(x^2y+xy)}_{F}-\underbrace{6}_{F}\underbrace{x}_{F}\underbrace{y}_{F}\underbrace{(3x^2-1)^2}_{F}$
Local (L3)	$3y-3(\underbrace{x^2y}_{T}+\underbrace{xy}_{T})-6xy(\underbrace{3x^2}_{T}-\underbrace{1}_{T})$
Local (L4)	$3y-3(\underbrace{x^2}_{F}\underbrace{y}_{F}+\underbrace{x}_{F}\underbrace{y}_{F})-6xy(\underbrace{3}_{F}\underbrace{x^2}_{F}-1)$

Tuesday/Thursday Classes: skip the rest.

9. Perform the indicated instruction.

   (a) Isolate the variable y in the expression

$$3y - 3(x^2y + xy) - 6xy(3x^2 - 1)^2 = 1$$

Follow the four steps to isolate.

$$3y - 3(x^2y + xy) - 6xy(3x^2 - 1)^2 = 1 \quad \text{Initial expression}$$

$$3y - 3x^2y - 3xy - \underbrace{6xy(3x^2 - 1)^2}_{\text{Don't expand just yet; } y \text{ is a factor for this term}} = 1 \quad \text{Expand using Dist. Law}$$

$$3y - 3x^2y - 3xy - 6xy(3x^2 - 1)^2 = 1 \quad \text{Collect terms with } y \text{ on one side}$$

$$y\left[3 - 3x^2 - 3x - 6x(3x^2 - 1)^2\right] = 1 \quad \text{Factor out } y$$

$$\boxed{y = \frac{1}{3 - 3x^2 - 3x - 6x(\underset{A}{3x^2} - \underset{B}{1})^2}} \quad \text{Divide both sides by factor attached to } y$$

$$y = \frac{1}{3 - 3x^2 - 3x - 6x(\underbrace{9x^4 - 6x^2 + 1}_{A^2 \; - \; 2AB \; + \; B^2})} \quad \text{Expand denominator for like terms, use } (A - B)^2$$

$$y = \frac{1}{3 - 3x^2 - 3x - 54x^5 + 36x^3 - 6x} \quad \text{Dist. Law}$$

$$y = \frac{1}{-54x^5 + 36x^3 - 9x - 3x^2 + 3} \quad \text{Like terms}$$

$$y = \frac{1}{-3(18x^5 - 12x^3 + x^2 + 3x - 1)} \quad \text{GCF}$$

$$\boxed{y = -\frac{1}{3(18x^5 - 12x^3 + x^2 + 3x - 1)}} \quad \text{Negative in front}$$

If you got to the first box that would be mostly correct! Last few steps are just because like terms could be created, so you should expand.

(b) Solve for all real solutions of x for the equation

$$2x^2 - 5x = -2$$

We factor and use zero product property.

$2x^2 - 5x = -2$	Initial expression
$2x^2 - 5x + 2 = 0$	Put in quadratic equation form
$(2x-1)(x-2) = 0$	new X method
$2x - 1 = 0 \quad , \quad x - 2 = 0$	Zero product property
$2x = 1 \quad , \quad \boxed{x = 2}$	Isolate x
$\boxed{x = \frac{1}{2}}$	Isolate x

$a = 2$
$b = -5$
$c = 2$

① For a

$$\begin{array}{c|c} 2 & 1 \\ 1 & 2 \end{array}$$

② For c

$$\begin{array}{c|c|c|c} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \end{array}$$

③ Check against b

$$\begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \rightarrow 2 \cdot 2 + 1 \cdot 1 = 5$$

needs to be negative!

close to positive b

$$\begin{array}{cc} 2 & -1 \\ 1 & -2 \end{array} \rightarrow 2(-2) + 1 \cdot (-1) = -4 - 1$$
$$= -5 \checkmark$$

(c) Solve for all real solutions of x for the equation

$$\frac{\frac{4}{x^2}-1}{x}=0$$

Deal with denominators. Similar to problem in lecture.

$$\frac{\frac{4}{x^2}-1}{x}=0 \qquad \text{Initial expression}$$

$$x\cdot\frac{\frac{4}{x^2}-1}{x}=0\cdot x \qquad \text{Multiply by LCD, } x$$

$$\cancel{x}\cdot\frac{\frac{4}{x^2}-1}{\cancel{x}}=0 \qquad \text{Frac. Prop. 5}$$

$$\frac{4}{x^2}-1=0 \qquad \text{Rewrite for better optics on our problem}$$

$$x^2\left(\frac{4}{x^2}-1\right)=0\cdot x^2 \qquad \text{Multiply by LCD, } x^2$$

$$x^2\cdot\frac{4}{x^2}-x^2\cdot 1=0 \qquad \text{Dist. Law}$$

$$\cancel{x^2}\cdot\frac{4}{\cancel{x^2}}-x^2=0 \qquad \text{Frac. Prop. 5}$$

$$4-x^2=0 \qquad \text{Rewrite for better optics on our problem}$$

$$4=x^2 \qquad \text{Get in form } x^2=c$$

$$\pm\sqrt{4}=\sqrt{x^2} \qquad \text{Take square root}$$

$$\boxed{x=\pm 2} \qquad \text{Done}$$